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WHOLESALE PROVISIONING MODELS:
MODEL DEVELOPMENT

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Alternative wholesale provisioning models for secondary items are developed. The models allocate optimally a given provisioning budget with such objective functions as units short, supply material availability, mean supply response time and availability. The models allow the inclusion of essentiality weights to discriminate component criticality. A procedure is also discussed for determining the minimum budget required to satisfy a specified level of performance.

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ABSTRACT

In this report we develop alternative wholesale provisioning models for secondary items with an aim toward use by both the Ships Parts Control Center (SPCC) and the Aviation Supply Office (ASO). General background into the wholesale provisioning problem is given, and the present provisioning models in use at SPCC are summarized. We review the demand forecasting problem, and we address the issue of demand distribution assumptions. Then various alternative wholesale provisioning models are developed. These models attempt to allocate optimally a given provisioning budget with such objective functions as (1) units short, (2) essentiality-weighted units short, (3) essentiality-weighted requisitions short, (4) supply material availability (SMA), (5) time-weighted units short, (6) essentiality-weighted mean supply response time, and (7) system availability. We conclude with theoretical comments about the alternative models and discussion of a companion problem that can be solved to determine the minimum budget required to attain a specified level of performance.

This is the first of three reports on the wholesale provisioning problem. The second report will present results of a numerical evaluation of the proposed alternative models and the current model. The third report addresses the issue of phased provisioning and offers a heuristic method for determining how to phase budget expenditures over time.

1. INTRODUCTION

In the spring of 1982, the Naval Supply Systems Command (NAVSUP) asked the Naval Postgraduate School to develop improvements to the existing peacetime wholesale provisioning models for secondary items used by the Ships Parts Control Center (SPCC) and the Aviation Supply Office (ASO). This effort was motivated by NAVSUP's Resolicitation Project, the major objective of which is to acquire new computer hardware. However, it also provides an opportunity to take a hard look at existing models in use by NAVSUP's Inventory Control Points (ICP) at SPCC and ASO and to make changes as desired before the ICP software is installed in the new computers. These changes should include wholesale and retail model improvements, both in provisioning and replenishment of secondary items . The most important questions to be answered by the Naval Postgraduate School are:

1. Do the existing models provide the best supply support for the Navy's weapon systems?
2. How should those models which do not provide the best support be changed?

In responding to these questions, it makes sense to begin with "the beginning" of supply support-- provisioning for a weapon system--because, once an improved provisioning model is developed, the replenishment models can be modified in such a way as to provide a smooth transition to continued wholesale supply support. This report will therefore concentrate on the provisioning models.

Wholesale supply support provides backup or "systems" stock of repairable and consumable items for the retail levels. At present the retail levels use models such as COSALs (Coordinated Shipboard Allowance List) and AVCALS (Aviation Coordinated Allowance List). These are stocks of spare secondary items (items which may fail in use) designed to provide sufficient support for a ship deployment period of, say, 90 days. Whenever an item is used by a ship or a squadron, a requisition is immediately submitted to the wholesale system for a replacement. Thus, the COSALs and AVCALs serve only as emergency protection. All demands ultimately must be satisfied by the wholesale stocks.

The wholesale provisioning model provides protection until the wholesale replenishment model can initiate the first replenishment buy (or repair quantity for those secondary items which can be repaired) and that buy is manufactured and delivered. Thus, wholesale provisioning buy protection begins on the date of preliminary operational capability (POC) and continues for at least a replenishment procurement (or repair) leadtime.

Actually, it is expected that a wholesale replenishment buy will be initiated some substantial time after POC. This will occur when the reorder point established by the replenishment model is reached. The initial reorder point is determined differently by the two ICP's and it may be based on the initial expected demand which is predicted as part of the provisioning process. However, after six months, the reorder point begins to reflect the actual demands placed on the supply system. This may result in the value of the reorder point decreasing for a year or two.

The provisioning buy must be made early enough so that the stock will be on hand by the POC date or shortly thereafter. Thus it must be made at least a procurement leadtime prior to that date. Typical values of this initial leadtime appear to be between 2.0 and 2.5 years. With such long leadtimes the quality of forecasted installation schedules of a new system may be poor. And, when protection must be provided for a subsequent time which is at least as long as a replenishment leadtime, the quality of the forecasted demands over that time can also be poor.

Provisioning guidance is provided by the Department of Defense Instruction (DODINST) 4140.42 [7]. The authors of [7] were of the opinion that much stock of secondary items was procured which was not subsequently needed. Thus, in an attempt to remedy this perceived situation, a model was developed which would provide only a very conservative quantity of wholesale stock for those items which were new to the supply system. For items already stocked in the system, they allowed no additional stock to be procured specifically for a new system. However, if the anticipated increase in demand was sufficiently large, it could be entered into the demand forecasting process by the inventory manager and its influence could trigger an earlier replenishment buy than would have occurred without it.

The authors of [7] did appreciate that improved models for provisioning new items would evolve and stated that:

> Inventory optimization models may be used as a basis for stockage in lieu of the model in this Instruction if the following criteria are met: (1) An optimization technique, developed to minimize system downtime or time-weighted requisitions short is used. (2) No lower limits are placed on requirements that will result in stockage as a demand-based item, of an item that would not be stocked without the lower limits. (3) A monetary base point is determined indicating the approximate value of the requirement in accordance with the instructions contained in enclosure 2. The second criterion may be omitted if only those items meeting the stockage criteria in accordance with this Instruction are considered in the optimization model. The longest applicable Program Forecast Period may be used in the optimization model, after the monetary base has been established.

Such improvements have indeed been developed by NAVSUP and are in use at both SPCC and ASO. However, as was stated above, NAVSUP is interested in developing even better models. This report represents the first step and will concentrate on a model to improve the initial supply support for new secondary items that will be needed. The issue of how to handle items already stocked will be the subject of a later report.

This report begins with a discussion of the existing wholesale provisioning model used by SPCC since it is more mathematically sophisticated than the ASO model and it is also easier to understand. Next, the demand process is examined and several alternative models intended for use by both SPCC and ASO are developed. These alternative models develop a budget using the COSDIF process and then allocate that budget so as to optimize system performance with respect to such measures as supply material availability, mean supply response time, and availability. A discussion of some of the theoretical properties of the models is provided and recommendations based on those properties are made.

Three appendices are included. The first presents the details of the forecasting models from [7] and from the SPCC and ASO models. The second discusses the cost difference or COSDIF formula which is required by [7] for developing the budget constraint. The last examines a widely held misconception about the relationship between mean supply response time and availability.

This is the first of three reports on wholesale provisioning. The second report describes the computational algorithms used for provisioning with each of the alternative models and provides some numerical evaluations of the models. The third report examines the issue of phased provisioning and offers a heuristic procedure for determining how to divide a provisioning budget over a period of time.

2. THE CURRENT SPCC PROVISIONING MODEL

DEVELOPMENT OF THE BUDGET CONSTRAINT

The model that SPCC uses to determine initial wholesale stocks for new items is a variant of the model proposed in reference [7]. The application/operation which implements the model is designated as D55 [19]. The model makes range and depth decisions separately for each item in the system for the purpose of developing a budget constraint.

The initial phase of the process is to develop a budget constraint. The first step of this phase is the determination of a range of items to be considered for stockage. This is done by computing the COSDIF value for each item. The purpose of the computation is to decide if an item is to be stocked as a demand-based item. The COSDIF value is obtained from a formula which was stated in reference [7]. The current version of COSDIF used by SPCC is presented in reference [19] and is reproduced in Appendix B of this report. Appendix B also presents the motivation for the formula and a critique of the procedure. Elements of the COSDIF include the expected costs of stocking the item and having no demands over two years, the expected costs of stocking an item and experiencing demand for one year, and the expected costs of not stocking an item for one year and having to make spot buys when demands occur. The difference between the sum of the first two elements and the last element is the "cost difference" or COSDIF. If the value of the COSDIF is negative then the item will be stocked; otherwise it will not be stocked insofar as budget development is concerned.

If an item has been determined by the COSDIF as appropriate to stock, its depth of stockage is next computed. The depth is taken to be equal to the demand expected over a period of time consisting of the forecasted replenishment procurement leadtime plus one quarter. The extra quarter of demand is viewed as safety stock. The procedure used to develop the initial demand prediction is discussed in Appendix A.

Those items which fail to satisfy the COSDIF criterion are next re-examined to determine if they are coded as insurance or numeric stockage objective (NSO) items. If so, the depth of each item is taken to be one minimum replaceable unit (MRU).

The cost of buying the amount of each item to the depth specified above is then determined, and the total dollar value of a provisioning package is obtained by summing these procurement costs over all of the NSO items, the insurance items, and those items passing the COSDIF test. This sum is then established as "the budget constraint" in accordance with reference [7]. The dollar value so determined is considered as a firm upper limit on the amount of money that can be spent to purchase those items that are to be stocked.

The actual range and depth of items that must be stocked, however, may deviate from the values used to determine the budget. As was noted in chapter I, an alternative provisioning model may be used to implement the range and depth provided the budget constraint as determined above is satisfied and the model objective is consistent with minimization of system down time or time-weighted requisitions short. The determination of the budget constraint is therefore an important element.

THE VARIABLE THRESHOLD MODEL FOR BUDGET IMPLEMENTATION

SPCC has developed a model, called the Variable Threshold Model, for making the budget implementation. In that model, all new items are considered to be candidates for stockage and are re-evaluated based on unit cost and the expected demand during the procurement leadtime.

The first step of the model is to compute the variable threshold value of each item. This value is computed using the variable threshold formula:

$$V(i) = \left[1 - \exp(-D_i)\right] / C_i$$

where C_i is the unit cost of the item and D_i is the expected demand during the procurement leadtime (see Appendix A). The variable threshold value is the mechanism used by SPCC to rank the items in a provisioning package according to desirability of stockage. Those items having a high value are placed at the top of the list. The numerator of the threshold value is the probability of one or more demands during a procurement leadtime when the demand is Poisson distributed. It comes from an analysis by Burton and Jaquette [2] in which the term represented the decrease in the expected number of backorders at a random time resulting from an increase in units stocked from zero to one for an item that has Poisson distributed demands. Reference [2] addressed only very low-demand items which are currently viewed as insurance or NSO items.

The variable threshold formula is a typical "benefit-to-cost" ratio used in the first step of a marginal analysis allocation scheme. Similar expressions are found in the alternative models which we present later.

The next step is to compute the depth for each item based on a risk formula similar to that used in the replenishment models. The formula is

$$RISK = IC / (IC + \lambda_s E)$$

where IC is the annual holding cost, λ_s is the shortage cost, and E is the item essentiality.

Let H(x) represent the probability that the leadtime demand for a given item is greater than or equal to x. Then, the depth for the item is taken to be the smallest value of x such that H(x) is not greater than the calculated RISK value. Thus, if x_i* represents the depth for item i,

$$x_i* = \min x;$$

$$\text{such that } H_i(x) \leq RISK = \frac{IC_i}{IC_i + \lambda_i E_i}$$

The actual probability distribution used for the calculation depends on the expected annual demand frequency. The Poisson distribution is used if the expected annual demand is less than or equal to one; the normal distribution is used when the expected annual demand or procurement leadtime demand is 20 or more; and the negative binomial distribution is used when the expected annual demand is between one and 20. The actual depth is constrained to be no more than two years of expected demand if the item is consumable and no more than the expected "net" attrition demand during a procurement leadtime plus one quarter if the item is repairable (see Appendix A).

The final step is to allocate the budget constraint among the items. The part of the total budget required to purchase the MRU's for the insurance and NSO items is reserved for these items and is subtracted from the total budget available. The remaining dollars are next allocated by successively purchasing the depths determined above for the items ranked in order of highest to lowest variable threshold value. The item with the highest variable threshold value is purchased in the depth specified above; the remaining budget is decremented; the item with the second highest variable threshold value is purchased next. This procedure continues until the budget is depleted or until every item is purchased at the specified depth. Any items already designated as insurance or NSO items that qualify for stockage under the above procedure will first have the depth quantities reduced by the already funded MRU's.

A comparison of the SPCC budget allocation approach with that described in reference [7] was done by the Fleet Material Support Office (FMSO) and reported in reference [9]. The results of the study showed that the variable threshold model provided more range and less depth than did the model of reference [7] for the same budget amount.

3. MODEL BACKGROUND

INTRODUCTION

The provisioning model currently in use at SPCC was discussed in the previous chapter. That model is a variant of the model proposed in DODI 4140.42. The provisioning procedure described in DODI 4140.42 is an aggregate of the Army's replenishment range model (COSDIF) and an attempt to conservatively forecast leadtime demand (under the assumptions that the components have reliabilities equal to their design values). That instruction provides uniform guidelines for the military services under which the services could develop their own provisioning models for carrying out the budget implementation The instruction does, however, allow only certain types of models. Paragraph IV.D of the instruction states:

> "Mathematical models may be used which may provide for a different mix of inventory than that stated herein, providing a financial base is established in accordance with the policies outlined in this instruction and an objective to minimize system downtime or time-weighted requisitions short is included in model."

We examine models in this report which focus on these types of objectives. In particular, we develop models which optimize with respect to:

1. expected number of stockouts
2. expected time-weighted units (requisitions) short
3. mean supply response time, and
4. availability.

Before presenting these models, it is appropriate to examine some background issues which affect all of the models that follow.

THE PROVISIONING INTERVAL

There are four key dates for consideration in the provisioning problem:

1. POC: the date of planned operational capability (part of the provisioning package should be in place by this date),
2. POC - PCLT: a procurement leadtime prior to POC,
3. POC + TR: the time beyond POC at which the first replenishment buy is made, and
4. POC + TR + PCLT: the time at which the first replenishment buy is received.

The primary concern of provisioning is to determine the number of units of each component of a system to be purchased at time POC - PCLT (and possibly purchase at additional times between POC - PCLT and POC). In order to reduce the likelihood of stockouts, the provisioning buy made at time POC - PCLT should be sufficiently large to satisfy anticipated demands in the interval (POC, POC + TR + PCLT). If good forecasts of demand during that interval and of the times TR and PCLT were available, the solution of the provisioning problem would be fairly routine. However, the provisioning problem is characterized by a great deal of uncertainty with respect to the failure rates (BRF'S) of new equipments, the procurement leadtime, the time TR, and even the installation schedule of new equipments.

During the time interval from POC to POC + TR + PCLT, the population of equipments will grow as additional units are installed in the operational community. This will cause the aggregate failure rates to increase over the time interval of concern in the provisioning problem. Actual installation schedules are subject to a great deal of uncertainty; the BRF estimates often represent only guesses at the failure rates since there are little or no operational data from which to estimate the failure rates; and the time lags are very large (the time interval from POC - PCLT to POC + TR + PLT may be on the order of four years or more). Thus it is not surprising that the provisioned quantities frequently don't match the demands very well. Faced with such uncertainties, no analytical model will cure all of the provisioning ills. However, the use of rational provisioning procedures should minimize such problems and, in addition, lead to improved replenishment models.

PROVISIONING OBJECTIVE FUNCTIONS

A key ingredient of DODI 4140.42 is the budget constraint for the procurement of items to be provisioned. No flexibility exists in the method used to determine the budget. (The actual values of the parameters used in the COSDIF formula can, however, be selected by the users.) As was noted at the start of this chapter DODI 4140.42 does allow limited flexibility in the type of provisioning model used to allocate the given budget among new items being considered. It can either minimize time-weighted units short or minimize system downtime. However, models such as the variable threshold which actually do neither have been approved. As a consequence, many measures of system performance have been and continue to be considered for objectives of provisioning. Some of the more popular are availability, probability of a stockout, fill rate, backorder accumulation, system downtime, time-weighted requisitions short, essentiality-weighted shortages, mean supply response time, and inventory costs. The use of costs as the principle measure of performance is complicated by the impossible task of quantifying such elements as stockout costs and obsolescence costs. The use of true system availability models for the wholesale level is complicated be the need for information at the operational level on a system-by-system basis. That information is currently not available. Such models are therefore not presently feasible for the wholesale level.

Some implementations of availability models for stockage decisions define availability for individual components (as opposed to weapon systems) in terms of the ratio of mean time between failures (MTBF) to the sum of MTBF, mean time to repair (MTTR), and mean supply response time (MSRT) (or other similar expressions); see, for example, references [3] and [23] . The proponents of such availability models argue that MSRT is the only term which pertains directly to the provisioning stockage decision. Consequently, these availability models really attempt to minimize MSRT. (Appendix C provides a simple example to show that minimization of MSRT does not necessarily yield the same solution as maximization of system availability.)

FORM OF THE MODELS

In all of the models which we develop, the optimization problem is of the form:

minimize (maximize) $\sum_i f_i(s_i)$

subject to $\sum_i c_i s_i \leq B.$

where $f_i(s_i)$ is the performance level for item i when s_i units are stocked, c_i is the unit price of item i, and B is the total amount budgeted for the provisioning package. The form of the optimization problem implicitly assumes separability; that is, the total system performance can be separated into a function which is the sum of the performance generated by each of its components. This form may not always be appropriate because it ignores the system configuration and any interaction between components. However, we feel that such an assumption is necessary to assure mathematical tractibility and implementation feasibility. It is also in keeping with the piece-parts support view which is traditional in the supply system. True "systems support" models are still a long time away.

When the performance functions are separable and well behaved the mathematical solution to the optimization problem can readily be obtained using such straightforward techniques as the generalized Lagrange multiplier approach of Everett [8] or marginal analysis [12].

RANGE RULES

All of the models which we will address solve simultaneously the range and depth problems. There is no explicit range rule. Instead, zero depth is equivalent to a no-stockage decision. Insurance items and numeric stockage objective (NSO) items will have to be handled separately from demand based items just as they are today. For the items which are demand based the models require an assumption about the distribution of demand. The next chapter supports the use of the Poisson probability distribution with an expected demand rate which depends on the BRF estimate and a time-weighted installed population (TWAMP).

4. THE DEMAND DISTRIBUTION

One of the most critical elements in any stockage model is the estimation of the total number of failures (demands) for a component which will be generated during the provisioning protection interval. The proper way to handle demand uncertainty is to model the demand process as a random process with a probability law from a known family of probability distributions and having parameters which are estimated from the specific information that is available for each item. In the provisioning problem, this information is pretty skimpy, consisting essentially of contractor provided estimates of failure rates (TRF's) and the schedule of installations.

This chapter discusses the probability distribution that should be used for the provisioning stockage decisions. We show that, under the assumption of a constant failure rate, the total demand for a given item over any time interval is Poisson distributed with parameter λ which depends on the time-weighted average month's program (TWAMP).

Consider a single component with constant failure rate α during the interval $(0,T]$. Let N be the total number of installations of the component during $(0,T]$, and let $0 = t_1 \leq t_2 \leq \ldots \leq t_N$ be the installation times of the N units of the component. Let X_i be the random number of failures of unit i in $(0,T]$. Then, because of the constant failure rate, the distribution of X_i is Poisson with parameter $\alpha(T-t_i)$ (see, for example, Ross [21]); i.e.,

$$P(X_i = x) = (\alpha(T-t_i))^x \exp(-\alpha(T-t_i)) \;/\; x! \qquad x=1,2,\ldots$$

Let us now consider the total number of demands generated by the i^{th} and j^{th} installed units. If we assume that the failures of the i^{th} and j^{th} units are independent, the probability distribution of the sum of X_i and X_j is given by the convolution of the distributions of X_i and X_j. Thus,

$$P(X_i + X_j = m) = \sum_{k=0}^{m} P(X_j = m-k)P(X_i = k)$$

$$= \sum_{k=0}^{m} \frac{(\alpha(T-t_j))^{m-k} \exp(-\alpha(T-t_j))}{(m-k)!} \frac{(\alpha(T-t_i))^{k} \exp(-\alpha(T-t_i))}{k!}$$

$$= \frac{\alpha^m(2T-t_i-t_j)^m \exp(-\alpha(2T-t_i-t_j))}{m!} \sum_{k=0}^{m} \binom{m}{k} \left(\frac{T-t_j}{2T-t_i-t_j}\right)^{m-k} \left(\frac{T-t_i}{2T-t_i-t_j}\right)^{k}$$

If we let $p = (T-t_i) / (2T-t_i-t_j)$, then $1-p = (T-t_j) / (2T-t_i-t_j)$ and the above summation is recognized to be the sum of binomial probability masses. Hence, that sum is unity, and the distribution of the sum of X_i and X_j is Poisson with parameter $\alpha(2T-t_i-t_j)$. Let $D = X_1 + X_2 + \ldots + X_N$ be the total demand over all installed units during the interval $(0,T]$. The above result is easily extended to show that D is Poisson distributed with parameter $\alpha(NT-\sum_{i=1}^{m} t_i)$. Let $f(t)$ be the total number of installations at time t. Then $f(t)$ is a step function with jumps at times $t_1, t_2, \ldots, t_N$. The integral of $f(t)$ over the interval is therefore $\sum_{i=1}^{N} (T-t_i) = NT - \sum_{i=1}^{N} t_i$. But this integral when normalized by T is the definition of the TWAMP. (TWAMP as defined in Appendix A treats time as discrete in units of months and therefore must make certain adjustments to accomodate installations occuring during a month. The differences are minor). Thus, the total demand over the program time base $(0,T]$ is Poisson distributed with parameter αT*TWAMP. Figure 4.1 illustrates this for the case in which 9 total installations occur over the interval $(0,T]$.

If all of the installations took place at POC ($t_1 = t_2 = \ldots = t_N = 0$), the process describing the total demand is a homogeneous Poisson process with rate $\lambda = N\alpha$. However, because of the phased installations, the total demand process has a mean rate which is a nondecreasing function of time; i.e., the demand process is a non-homogeneous Poisson process. The mean rate of demand at time t is $\lambda(t) = \alpha \int_0^t f(t)dt$.

In the next chapters we develop alternative models for determining the amount of stock to be provisioned for each component of a provisioning package. In all of those models we assume a Poisson distribution for the total demand for each item. The parameters λ_i referred to in those models will be assumed to be the "TWAMP'ed" values discussed above.

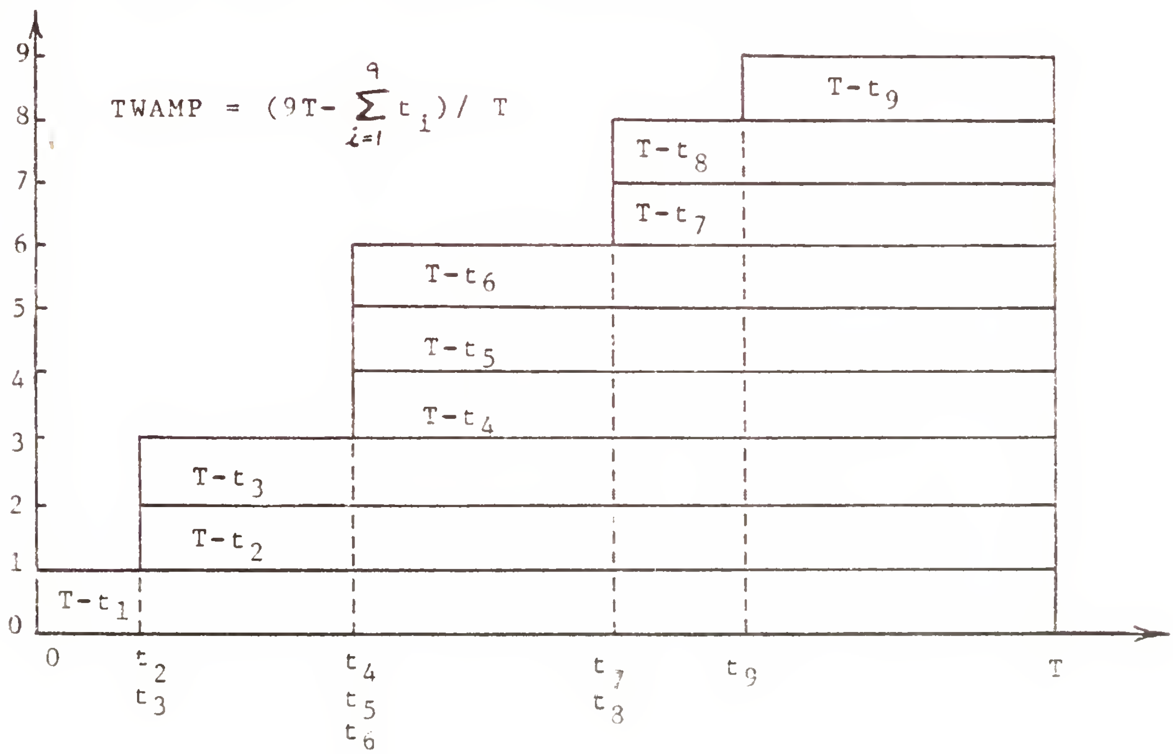

Figure 4.1: Time-Weighted Installations

5. UNITS SHORT MODELS (SMA)

INTRODUCTION

Consider a weapon system composed of n components. Let $s_i, s_2, \ldots, s_n$ be the number of units of components 1 through n, respectively, to be provisioned. Let c_i be the unit cost of component i and let B be the total budget available to fund spares for the provisioning package. (B will be determined by the COSDIF process.) As discussed earlier, the provisioning problem is interested in the time interval (POC, POC + TR + PCLT). However, for notational convenience we will refer to the interval as $(0, T_i]$. Let $X_i(T_i)$ be a random variable describing the total number of demands for component i in the interval $(0, T_i)$; $p_i(x) = \mathrm{Prob}[X_i(T_i) = x]$; $P_i(x) = \mathrm{Prob}[X_i(T_i) \leq x]$; and $H_i(x) = \mathrm{Prob}[X_i(T_i) \geq x]$. As indicated in the previous chapter, we assume that

$$p_i(x) = \frac{(\lambda_i T_i)^x \exp(-\lambda_i T_i)}{x!}$$

where $\lambda_i T_i$ is the total expected demand over $(0, T_i]$, and λ_i is estimated from the technical replacement factor (TRF) and the installation schedule.

MODEL I--UNITS SHORT

The first model considers the objective function to be minimization of the expected number of stockouts over the interval $(0, T]$. Thus, we want to find those integer values $s_i, s_2, \ldots, s_n$ which minimize the total expected units short subject to a constraint on the total procurement funds. If s_i units of component i are stocked and x_i units are demanded, then the total number of units short for component i is

$$\begin{cases} x_i - s_i & \text{if} \quad x_i > s_i \\ 0 & \text{otherwise.} \end{cases}$$

The expected number of units short for component i is therefore

$$\sum_{x_i=s_i+1}^{\infty} (x_i - s_i)\, p_i(x),$$

and the optimization problem is:

$$\text{Minimize } Z(S) = \sum_{i=1}^{n} \sum_{x_i=s_i+1}^{\infty} (x_i - s_i)\, p_i(x_i), \qquad (5.1)$$

subject to $\sum_{i=1}^{n} c_i s_i \leq B,$

and s_i = nonnegative integer.

This constrained nonlinear separable optimization problem can be solved using the generalized Lagrange multiplier approach. Let S be the n-vector $(s_i, s_2, \ldots, s_n)$, and let $L(S;\theta)$ be the Lagrangian function:

$$L(S;\theta) = \sum_{i=1}^{n} \sum_{x_i=s_i+1}^{\infty} (x_i - s_i) p_i(x_i) - \theta(B - \sum_{i=1}^{n} c_i s_i).$$

Because of the separability, this can be rewritten as

$$L(S;\theta) = \sum_{i=1}^{n} L_i(s_i) - \theta B;$$

where

$$L_i(s_i) = \sum_{x_i=s_i+1}^{\infty} (x_i - s_i)p_i(x_i) + \theta c_i s_i .$$

If the demand distributions were continuous, the optimal solution to (5.1) would be given by solving the $n+1$ simultaneous equations for s_i^*, $i = 1$ to n, and θ^* which satisfy [12]:

$$\frac{\partial L_i}{\partial s_i} = 0 \ ; \quad i = 1,2,\ldots,n,$$

$$\frac{\partial L}{\partial \theta} = 0.$$

Because of the integer nature of the decision variables, the above procedure has to be modified slightly. Let

$$\Delta L_i(s_i) = L_i(s_i) - L_i(s_i - 1)$$

be the change in the function L_i resulting from an increase in the stockage level for component i from s_i-1 to s_i. The optimal value of s_i is the largest value of s_i such that $\Delta L_i(s) < 0$. Simplifying, we determine that

$$\Delta L_i(s_i) = \theta c_i - H_i(s_i)$$

Therefore, s_i^* is the largest nonnegative integer such that

$$H_i(s_i^*) > \theta^* c_i \qquad i = 1,2,\ldots,n. \qquad (5.2)$$

The above simultaneous equations cannot be solved in closed form for the decision variables. However, the solution can be obtained easily using the approach by Everett (Reference [8]). Select a trial value for θ and find

$s_i^*(\theta)$ from equation (5.2) for $i = 1,2,\ldots,n$. Then evaluate

$$B(\theta) = \sum_{i=1}^{n} c_i s_i^*(\theta)$$

If $B(\theta)$ is close to B (within prespecified tolerance limits) the process stops and $s_i^* = s_i^*(\theta)$. If $B(\theta)$ is not close to B, a new value of θ is selected and the process repeats.

Aid in the selection of the new values of the multiplier is available from the theory of the generalized Lagrange multiplier approach presented by Everett [8]. The multiplier θ has the mathematical interpretation of being

$$\theta = -\frac{\partial Z}{\partial B} .$$

Thus θ can be thought of as a measure of the decrease in the total units short per additional budget dollar. Since the objective function is a monotone non-increasing function of B, θ must always be nonnegative. If the amount of a current solution does not use up all of B then decrease θ to increase the budget expenditures; if it exceeds the budget then increase θ to decrease the expenditures.

The theory also provides guidance at each iteration as to the quality of the incumbent solution. A lower bound on the optimal value of the objective function is provided (Daeschner [5]) at each iteration from the inequality

$$Z(S^*) \geq Z(S^*(\theta_j)) - \theta_j(B - B(\theta_j))$$

Comparison of the present value of Z; namely, $Z(S^*(\theta_j))$, with the greatest lower bound yet experienced (for cases where the budget constraint was satisfied; i.e., the amount used is $B(\theta_j) \leq B$) will provide information as to the maximum benefit to be gained by continued searching. For example, suppose that the first three iterations yielded values of the above lower bound of (1003, 1155, and 1167). Also, suppose that the current value of the objective function is $Z(S^*(\theta_j)) = 1168$. We know that the optimal solution cannot be smaller than 1167, so there is little to be gained by continued searching.

The iterative process described above will determine the values of the decision variables which will minimize the total expected units short subject to the given budget constraint. Exact equality of budget expenditures may be impossible to obtain because of the integer nature of the variables. Therefore, the result determined may not actually be optimal, but the difference is not likely to be significant. It is important to appreciate that, for a given budget expenditure $B(\theta)$, the solution obtained is optimal.

MODEL II--MODIFIED UNITS SHORT

The model of the previous section minimized the total number of expected units short for the provisioned system subject to the budget constraint. It is a trivial matter to modify that model to allow for weighting the components by essentialities. Thus, consider the objective function

$$Z_2(S) = \sum_{i=1}^{n} \sum_{x_i=s_i+1}^{\infty} E_i(x_i - s_i)\, p_i(x_i)$$

obtained by weighting the units short for component i by the essentiality code E_i where $E_i > E_j$ if component i is judged to be more essential than component j. The minimization of $Z_2(S)$ subject to the budget constraint gives a formula for finding the optimal solution which is similar to that found in the previous section. The optimal stockage level for item i is the largest nonnegative integer s_i such that

$$H_i(s_i) > \frac{\theta c_i}{E_i}$$

Similarly, the units short objective function can be modified to minimize the total expected requisitions short. Let g_i be the expected quantity demanded per requisition for component i, and consider the objective function

$$Z_3(S) = \sum_{i=1}^{n} \sum_{x_i=s_i+1}^{\infty} \frac{(x_i - s_i)}{g_i} p_i(x_i))$$

obtained by dividing the expected units short by the expected requisition size to yield the expected requisitions short. For this problem, s_i^* is the largest nonnegative integer such that

$$H_i(s_i) > \theta c_i g_i$$

The procedure used to search for the value of θ for these two modifications is the same as that discussed for Model I.

MODEL III--SMA

To conclude this chapter on units-short models we next consider an objective function related to those described above, but which is stated in terms more closely related to presently reported measures of supply effectiveness; namely, supply material availability (SMA).

SMA is defined as the fraction of all requisitions that are satisfied by stock on hand over a given time period. It would seem reasonable to attempt to maximize the predicted SMA for the items in a provisioning package subject to a constraint on the total procurement investment.

Let D_i be the total demand for item i during the provisioning interval $(0,T_i]$. We assume that requisitions are for one unit. Let F_i be the number of demands satisfied, and let U_i be the number of demands unfilled. Then, clearly,

$$D_i = F_i + U_i$$

and the expected number of demands satisfied (fills) is

$$E(F_i) = E(D_i) - E(U_i) = \lambda_i T_i - E(U_i)$$

Summing over the n items in the provisioning package, we get the total expected number of "fills" to be

$$\sum_{i=1}^{n} E(F_i) = \sum_{i=1}^{n} \lambda_i T_i - \sum_{i=1}^{n} E(U_i)$$

Division by $\sum_{i=1}^{n} \lambda_i T_i$ yields the ratio of expected fills to expected demands (SMA).

$$SMA = \frac{\sum_{i=1}^{n} E(F_i)}{\sum_{i=1}^{n} \lambda_i T_i} = 1 - \frac{\sum_{i=1}^{n} \sum_{x_i=s_i+1}^{\infty} (x_i - s_i) p_i(x_i)}{\sum_{i=1}^{n} \lambda_i T_i} \tag{5.3}$$

Observe in (5.3) that the denominator does not depend on the decision variables and the summation term in the numerator in the total expected number of units short. Thus, the allocation of a given budget which minimizes expected units short will be the same as the allocation which maximizes SMA. However, since the value of SMA is likely to be more operationally meaningful than the total expected units short, the performance prediction should probably be stated in terms of SMA. This will aid in the communication of model results.

Just as with the units short model, the SMA model can be modified to include essentiality weights. All that is required is to weight the units short by the essentiality values and to modify the denominator to include also the essentiality values. The essentiality-weighted SMA (ESMA) would then be:

$$\mathrm{ESMA} = 1 - \frac{\sum_{i=1}^{n} E_i \sum_{x_i=s_i+1}^{\infty} (x_i - s_i) p_i(x_i)}{\sum_{i=1}^{n} E_i \lambda_i T_i}$$

This overall value can also be written in terms of the ESMA's for the individual items. Let $\mathrm{SMA}_i(s_i)$ be the expected SMA value for item it when s_i units are stocked. Then ESMA can be written as follows:

$$\mathrm{ESMA} = \frac{\sum_{i=1}^{n} E_i \lambda_i T_i \cdot \mathrm{SMA}_i(s_i)}{\sum_{i=1}^{n} E_i \lambda_i T_i}$$

This alternative form illustrates that the overall ESMA value is a weighted average (weighted by essentialities and the expected number of demands) of the individual item SMA's.

SUMMARY

In this chapter we have considered models which allocate a given budget amount so as to optimize system performance with respect to units short or essentiality weighted units short. No consideration is given in these models to the length of time that must pass before a customer's requisition can be satisfied when a stockout occurs. This time delay is uppermost in the minds of the customers as it can directly affect the readiness of weapon systems. The models developed in the next chapter remedy this shortcoming by explicitly considering time-weighed units short.

6. TIME-WEIGHTED UNITS SHORT MODELS

We develop two models in this chapter which consider not only the shortages which accumulate over $(0,T]$, but also the length of time that each shortage exists. Classical inventory cost models recognize the effect that stockout delays have on system performance by imposing a stiff penalty cost for stockouts which grows as the stockout time increases. The models that we develop here do not assess penalty costs since they are virtually impossible to quantify. Instead, the objective function is the direct minimization of expected time-weighted units short (TWUS). Such an objective function is appropriate when the readiness of the operational units suffers as long as replacement parts are unavailable. In a readiness measure such as operational availability, the time until a failed component is restored to serviceable condition is a key ingredient. When spares are available, that time is short. But when spares are not available, that time will be very long.

These models assume that if there is a spare item in the system then it is "immediately available" and that if there is no spare the demand becomes a backorder which is filled by the next order which arrives from the manufacturer.

MODEL IV--TWUS

Let us restrict attention initially to a single component. Let $X(T)$ be the random number of demands for the component in $(0,T]$. Let us assume that $X(T) = m$, and let

$$0 < T_1 < T_2 < \dots < T_m \leq T$$

be the times at which the m demands occur. If s units of the item are stocked, the total time-weighted units short, given $X(T) = m$ and $T_1, T_2, \ldots, T_m$, can be described by

$$\sum_{j=1}^{m-s} (T - T_{s+j}).$$

Figure 6.1 provides a graphic illustration of this derivation. The backorders are those demands giving negative net inventory. The length that each is outstanding is the time from when its demand occurs until the next order arrives at time T.

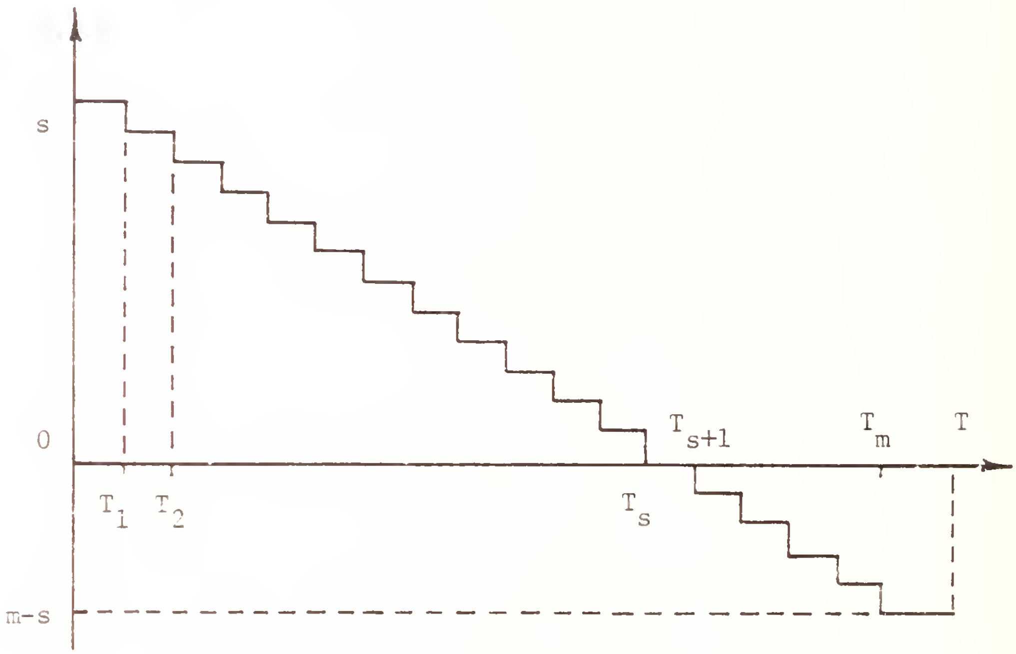

Figure 6.1: NET INVENTORY OVER TIME

The conditional expected value of the time-weighted units short is therefore:

$$E[TWUS(s) | X(T) = m] = \begin{cases} 0 & \text{if } m \leq s \\ \sum_{j=1}^{m-s} (T - E[T_{s+j} | X(T) = m]) & \text{if } m > s \end{cases} \tag{6.1}$$

Let us now determine $E[T_k | X(T) = m]$. Assuming a homogeneous Poisson process, the probability that the kth demand will occur between t and $t+dt$ can be determined as follows:

$$f_{T_k}(t|m)dt = \text{Prob}(t < T_k \leq t+dt | X(T) = m)$$

$$= \frac{P(k-1 \text{ events in } (0,t],\ 1 \text{ event in } (t,t+dt],\ m-k \text{ events in } (t,T])}{P(X(T) = m)}$$

$$= \frac{\dfrac{(\lambda t)^{k-1}\exp(-\lambda t)}{(k-1)!}(\lambda\ dt)\dfrac{(\lambda(T-t))^{m-k}\exp(-\lambda(T-t))}{(m-k)!}}{\dfrac{(\lambda T)^m \exp(-\lambda T)}{m!}}$$

$$= \frac{m!}{(k-1)!\ (m-k)!}\left(\frac{t}{T}\right)^{k-1}\left(\frac{T-t}{T}\right)^{m-k}\frac{dt}{T} .$$

Now, $E[T_k | X(T) = m] = \int_0^T t\ f_{T_k}(t|m)dt$

$$= \int_0^T t\ \frac{m!}{(k-1)!\ (m-k)!}\left(\frac{t}{T}\right)^{k-1}\left(\frac{T-t}{T}\right)^{m-k}\frac{dt}{T}$$

The change of variable, $u = t/T$, simplifies the above expression to

$$T\int_0^1 u\ \frac{m!}{(k-1)!\ (m-k)!}\ u^{k-1}\ (1-u)^{m-k}\ du$$

The integral is recognized to be the expected value of a beta distributed random variable with parameters $\alpha = k$ and $\beta = m-k+1$. Consequently, the conditional expectation that we seek is

$$T\left(\frac{\alpha}{\alpha+\beta}\right) = \frac{Tk}{m+1}$$

Returning to expression (6.1), we have determined the conditional expectation

$$
E[TWUS \mid X(T) = m] = \begin{cases} \sum_{j=1}^{m-s} T - T\left(\frac{s+j}{m+1}\right) & \text{if } m > s \\ \\ 0 & \text{if } m \le s \end{cases}
$$

Expanding the summation gives (for the case $m > s$)

$$
\frac{T(m-s)(m-s+1)}{2(m+1)}
$$

Finally, on summing over all possible values of m, the total expected time-weighted units short for the component is found to be

$$
E[TWUS(s)] = \sum_{m=0}^{\infty} E[TWUS(s) \mid N(T) = m] \; Prob[N(T) = m]
$$

$$
= \sum_{m=s+1}^{\infty} \frac{T}{2} \frac{(m-s)(m+1-s)}{m+1} \frac{(\lambda T)^m \exp(-\lambda T)}{m!}
$$

$$
= \frac{T}{2}\{(\lambda T)H(s) - 2sH(s+1) + \frac{s(s+1)}{\lambda T} H(s+2)\}
$$

$$
= \frac{T}{2}\{H(s+1)[\lambda T - 2s + \frac{s(s+1)}{\lambda T}] + p(s)(\lambda T - s)\} .
$$

Now that we have derived an expression for the expected time-weighted units short as a function of the stockage level s, let us write the optimization problem that we want to solve. Let $V_i(s_i)$ be the expected time-weighted units short for item i when s_i units are provisioned. We want to minimize the total essentiality weighted, time-weighted units short subject to a budget constraint. The Lagrangian statement of the problem is:

$$
\text{Minimize} \quad L(S;\theta) = \sum_{i=1}^{n} E_i V_i(s_i) - \theta[B - \sum_{i=1}^{n} c_i s_i].
$$

Let $L_i(s_i;\theta) = E_i V_i(s_i) + \theta c_i s_i$. As before, the optimal solution s_i^* is found by determining the largest s_i such that $\Delta L_i(s_i;\theta) < 0$ where θ is such that $\sum_{i=1}^{n} c_i s_i = B$. After simplification $\Delta L_i(s_i;\theta)$ can be written as:

$$\Delta L_i(s_i;\theta) = E_i[(1-P(s_i))(T_i - \frac{s_i}{\lambda_i}) + T_i p(s_i)] - \theta c_i$$

Thus, s_i^* is the largest s_i such that

$$(1-P_i(s_i))(T_i - \frac{s_i}{\lambda_i}) + T_i p_i(s_i) < \frac{\theta c_i}{E_i}.$$

We solve for the optimal values using exactly the same search (on θ) procedure as discussed in the previous chapter.

MODEL V--MSRT

One of the most widely reported measures of supply performance is the mean supply response time (MSRT). This measure inherits its popularity at least partly through the role it plays in the expression commonly used to define availability (see Chapter 7). In addition, MSRT is important by itself as an indicator of the success of the supply system in meeting response time goals. As a measure, it considers both the likelihood of satisfying demands from stock on hand and the length of the delay in satisfying demands when the system runs out of stock.

Let us determine the allocation of a fixed budget which will minimize the MSRT averaged over all of the items in a provisioning package. Let t be the time at which a failure occurs in $(0,T_i]$, and let $Y_i(t)$ be the total number of failures which have occurred from POC up to, but not including, t. Let $R_i(t)$ be the supply response time for the demand occurring at time t. We assume that $R_i(t)$ is a constant k_i (possibly zero) if $Y_i(t) < s_i$.

If the on-hand stock is not positive when the demand occurs at time t, then satisfaction of the demand will be delayed until k_i units of time beyond receipt of the first replenishment order which we assume will occur at time T. Thus,

$$R_i(t) = \begin{cases} k_i & \text{if } Y_i(t) < s_i \\ k_i + (T_i - t) & \text{if } Y_i(t) \geq s_i \end{cases}$$

The conditional mean supply response given the failure occurs at time t is

$$\begin{aligned} E[R_i(t)] &= k_i P[Y_i(t) < s_i] + (k_i + (T_i - t))P[Y_i(t) \geq s_i] \\ &= k_i + (T_i - t)P[Y_i(t) \geq s_i] \end{aligned}$$

The unconditional mean supply response time is found by integrating the above conditional expectation over $(0,T_i]$ with respect to the density of the time t. But, it is well known (see Ross [21]) that the time of failure of an arbitrary event generated by a Poisson process is uniformly distributed on the interval $(0,T]$. Thus, the density of t is

$$f_t(\tau) = \begin{cases} \frac{1}{T_i} & 0 \leq \tau \leq T_i \\ 0 & \text{otherwise} \end{cases}$$

and the mean supply response time is

$$\begin{aligned} MSRT_i(s_i) &= \int_0^{T_i} (k_i + (T_i - t_i)H_i(s_i)) \frac{1}{T_i}\, dt \\ &= k_i + \frac{1}{T_i}\int_0^{T_i} (T_i - t_i)H_i(s_i)dt \\ &= k_i + \frac{T_i}{2}H_i(s_i) - \frac{s_i}{\lambda_i} H_i(s_i+1) + \frac{(s_i+1)s_i}{2\lambda_i^2 T_i} H_i(s_i+2). \end{aligned}$$

Recalling the expression for the expected time-weighted units short we observe that

$$\mathrm{MSRT}_i(s_i) = k_i + \frac{V_i(s_i)}{\lambda_i T_i}$$

where $V_i(s_i)$ is the expected time-weighted units short when s_i units of item i are stocked. Finally, the aggregate mean supply response time for all items in the provisioning package is obtained by weighting the individual means by the expected cumulative demand:

$$\mathrm{MSRT} = \frac{\sum_{i=1}^{n} (\lambda_i T_i)\mathrm{MSRT}_i(s_i)}{\sum_{i=1}^{n} \lambda_i T_i}$$

$$= \frac{\sum_{i=1}^{n} k_i \lambda_i T_i + \sum_{i=1}^{n} V_i(s_i)}{\sum_{i=1}^{n} \lambda_i T_i} \qquad (6.2)$$

Since the denominator and the first summation in the numerator do not depend on the decision variables, s_1 through s_n, it is clear that the stockage levels which minimize the aggregate MSRT are the same as those that minimize the total expected time-weighted units short. As with the TWUS model, the MSRT model can be modified easily to incorporate essentiality weights. All that is needed is to replace the weights $\lambda_i T_i$ in (6.2) by $E_i \lambda_i T_i$.

Even though the TWUS model and the MSRT model will yield the same solutions, MSRT appears to be more operationally meaningful than is time-weighted units short. The next chapter develops a model which incorporates MSRT to yield another measure of performance of interest to the operational community; namely, operational availability.

7. AVAILABILITY MODEL

The final model which we develop as a candidate for provisioning is an availability model which incorporates as a key ingredient the MSRT discussed in the previous section. We believe that the Navy Supply System would be better served by using one of the previous models, but we include the availability model here to satisfy the present clamor for models which "spare to availability".

The commonly used definition of the availability for a single component is

$$A_i = MTBF_i/(MTBF_i + MTTR_i + MSRT_i)$$

where $MTBF_i$ is the mean time between failures, $MTTR_i$ is the mean time required to repair/replace a failed unit when a spare is available, and $MSRT_i$ is the mean supply response time (see References [3], [14], [15], [23]). Although this expression is actually a limiting result which is valid only under certain conditions ([13], [21]), it is useful for providing insights into the factors which affect availability and of the sensitivity of availability to those factors. For purposes of provisioning, we write the mean supply response time as a function of the total number of units provisioned, $MSRT_i(s_i)$, as was done in the previous section. The availability expression then becomes

$$A_i(s_i) = MTBF_i/(MTBF_i + MTTR_i + MSRT_i(s_i)) \tag{7.1}$$

Clearly, a primary objective of the Supply System should be to allocate resources so that weapon system availability is maximized. The key phrase in the previous sentence is "weapon system." This is where one of the major difficulties arises in trying to develop availability models. The availability of a weapon system is a very complicated function which depends on 1) the availabilities of its components, 2) the configuration of the weapon system (reliability block diagram), and 3) the way that the system is deployed operationally. Without making some simplifying assumptions, it would probably not be possible to develop any useful general purpose algorithms for maximizing weapon system availability.

Most existing availability models (see References [3] and [23], for example) make the following assumptions:

1) Component failures are generated by a Poisson process (lifetimes are exponential). This means that $MTBF_i = 1/\lambda_i$.
2) Component availabilities are given by expression (7.1).
3) The weapon system is configured with the components all in series.
4) The failure of one component is independent of the failures of the others.

These assumptions lead to the formula below for the availability of the weapon system.

$$A_o(s_1, s_2, \ldots, s_n) = \prod_{i=1}^{n} A_i(s_i) \tag{7.2}$$

The accuracy of the above expression depends on the extent to which the assumptions are valid. We doubt that they are all satisfied for any real weapon system, and we doubt that the resulting estimated weapon system availability would accurately reflect the actual observed system availability. As a consequence, we will refer to this model as the "pseudo-availability model." Nevertheless, as a model of system availability, (7.2) may be useful for the purpose of deciding the range and depth of items to be provisioned.

Consider the problem:

$$\underset{s_1,\ldots,s_n}{\text{maximize}} \quad A_o = \prod_{i=1}^{n} \text{MTBF}_i/(\text{MTBF}_i + \text{MTTR}_i + \text{MSRT}_i(s_i)) \tag{7.3}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} c_i s_i \leq B \qquad\qquad s_i = 0,1,2,\ldots$$

Taking the logarithm of both sides of the objective function, we get:

$$\ell n\, A(s_1, s_2, \ldots, s_n) = \sum_{i=1}^{n} \ell n(\text{MTBF}_i/(\text{MTBF}_i + \text{MTTR}_i + \text{MSRT}_i(s_i))).$$

Let $L(S_i;\theta) = \ell n(\text{MTBF}_i/\text{MTBF}_i + \text{MTTR}_i + \text{MSRT}_i(s_i)) - \theta c_i s_i$. Solving problem (7.3) is equivalent to solving the Lagrange problem:

$$\max L(s_1, \ldots, s_n;\theta) = \sum_{i=1}^{n} L_i(s_i;\theta) + \theta B.$$

The optimal solutions are given by determining the largest values of s_i such that

$$\Delta L_i(s_i;\theta) = L_i(s_i;\theta) - L_i(s_i - 1;\theta) > 0.$$

Simplification shows that s_i^* is the largest non-negative integer such that

$$\frac{J_i + \text{MSRT}_i(s_i - 1)}{J_i + \text{MSRT}_i(s_i)} \geq \exp(\theta^* c_i),$$

where $J_i = \text{MTBF}_i + \text{MTTR}_i$ and θ^* is that nonnegative Lagrange multiplier value such that $\sum_{i=1}^{n} c_i s_i^* = B$.

Unlike the previous models, it would not make sense to weight the individual components by their essentialities in the availability model. In fact, the series-system assumption is tantamount to an assumption of equal essentiality of components; i.e., the failure of any component will render the system unavailable. True availability models which explicitly consider system configuration automatically handle essentiality by consideration of the impact of each component failure on system performance (see Jee [13]). No external weighting or assignment of essentiality codes would be necessary. Such "true" availability models are, however, very difficult to develop and implement.

Because of the assumptions made in developing the pseudo-availability model in this section, it is unlikely that the system availability predictions provided by the model will be correct. Indeed, for even relatively small systems, the predicted availability will necessarily be very small. For example, a system composed of 25 components, each with an availability of 0.95, will have a predicted availability of only $(.95)^{25} = 0.2774$. Nevertheless, the budget allocation decisions made by the model may be very good.

To conclude this discussion about the pseudo-availability model we would like to comment on a claim frequently made in the availability literature (see, for example, [3] and [23]). Some of the availability analysts argue that when sparing to availability, the only term in the availability expression which involves the number of spares is MSRT. Therefore, they claim, maximization of system availability with respect to the number of spares for the components is equivalent to minimization of MSRT. A simple counterexample given in Appendix C shows that the claim is not true! However, for realistic choices of the various parameters, the allocations determined from the two models (MSRT and A_o) may be very much alike. The numerical evaluations which follow in the second report examine this issue.

8. COMMENTS ON THE MODELS

INTRODUCTION

The previous chapters have presented three basically different provisioning models:

(1) units short (SMA),

(2) time-weighted units short (MSRT), and

(3) pseudo-availability.

In this chapter we provide brief comments about the models as related to three areas:

(1) incorporation of a random protection interval,

(2) theoretical properties of the models, and

(3) solution of a related problem for use as a budgeting tool.

RANDOM PROTECTION INTERVAL

The previous models all assume that the length of the protection interval (POC to POC+T] is known. In reality, T is a random variable since both the procurement leadtime and the time from POC to the first replenishment buy are random variables. If one had information about the distribution of the random variable T, the previous models could be extended (at the cost of additional mathematical complexity) to accommodate the randomness of T.

Let $g_T(t)$ be the probability density function for T. What is required in the previous models to incorporate the randomness of T is to replace T everywhere by a dummy variable, say u, and to integrate, weighting by the density $g_T(u)$, over all possible values of u. One common assumption for the form of $g_T(u)$ is the gamma density. The gamma probability family has the proper range of values $(0$ to $\infty)$, and the family is extremely rich in the shapes that it can assume. Many theoretical and empirical studies have shown that the gamma distribution frequently approximates well the empirical distributions of leadtime. Furthermore, when the gamma distribution for leadtime is combined with the Poisson distribution for demand in an interval, the resulting distribution of leadtime demand is negative binomial (see Chapter 3 of [12]). This theoretical fact accounts for the selection of the negative binomial as one of three distributions used by the Navy ICP's in some of its wholesale provisioning and replenishment models.

For purposes of implementation in the provisioning models it is unlikely that any information will be available to adequately estimate the distribution of T. Also, since inclusion of a probability distribution for T increases the complexity of the models, we suggest that T continue to be treated as a constant. Of course, every effort should be made to estimate T as well as possible.

One of the elements of T is the length of time from POC to the time that the first replenishment buy is made. In addition to the elements already included in the models, this time period depends on an external element not yet considered--the replenishment reorder level. Given the size of the first provisioning buy, say s, and the size of the first replenishment reorder level r, it is easy to determine that the time from POC to the first replenishment buy is Erlang distributed (assuming a homogeneous Poisson process with rate λ generates demands) with parameters $n = s-r$ and λ. This is because the delay is the amount of time required to experience $s-r$ demands with exponentially distributed times between demands. The problem is that in order to integrate the provisioning and replenishment models it would be necessary to include the replenishment reorder level r as yet another variable in the provisioning model so that the size of the provisioning buy depends on r. Furthermore, the value of r in such an integrated model would logically depend on the provisioning stockage level so that solution of the model would require an iterative technique greatly complicating the model and increasing the computational burden. Thus, we suggest that the first replenishment buy not be triggered by the inventory position dropping below a reorder point, but be made automatically at a fixed time after POC, say $POC + \Delta t$. If the estimates of demand, installation schedules, and procurement leadtimes are accurate, it is reasonable that the first replenishment buy be made at POC (i.e., $\Delta t = 0$). Delaying the replenishment buy to some later date to allow the initial TRF estimates to be updated by actual demand data only lengthens the provisioning protection interval and increases the size of the required provisioning buy.

THEORETICAL PROPERTIES

The models developed in this report can be expected to lead to different solutions to the optimal budget allocation problem. In contrast to the Variable Threshold Model, each of the models also provides a prediction of the expected performance to be achieved from a given solution. Furthermore, some theoretical properties of the models can be examined to determine which model best reflects our intuitive notion about how the system is degraded by the presence of shortages, about the effect of uncertainty in our estimates of the demand rates, and about the propensity to stock items as a function of their demand rates, unit costs, essentialities, and leadtimes.

All of the models are more likely to stock the low cost, high demand-rate items, all other things the same. Weighting by essentialities is the only way offered here to oppose that tendency. Also, the models are all piece-parts oriented. No explicit consideration is given in any of the models for "weapon system effectiveness". However, essentiality weights can be used to reflect the importance of the individual items with respect to weapon system effectiveness. Proper assignment of essentiality weights should reflect what happens to the readiness of a weapon system if a shortage occurs in an individual item.

A COMPANION PROBLEM AS A BUDGETING TOOL

In the models which we have developed, we have assumed that a budget B was given at the outset. DODI4140.42 requires that the provisioning budget be determined by using the COSDIF model. As discussed in Appendix B, the COSDIF model first determines if an item is to be stocked by comparing the expected cost of stocking the item with the expected cost of not stocking the item. If the former cost is smaller, the item satisfies the range rule. The depth of a selected item is the expected demand during a procurement leadtime plus one quarter. The total "priced-out" value for all items satisfying the range rule is the provisioning budget (excluding the budget allowed for insurance and NSO items).

This procedure for determining the budget for a provisioning budget is cost-based requiring cost estimates that may not reflect accurately real costs. Furthermore, the COSDIF procedure does not provide any estimate of the performance that might be expected from the resulting budget. The alternative models presented in this report can each be rewritten in a mathematically-related form which could be solved to provide a tool useful for determining a provisioning budget.

We saw that each of the models described earlier could be written in the primal form:

$$\min(\max) \quad \sum_{i=1}^{n} f_i(s_i)$$

$$\text{subject to} \quad \sum_{i=1}^{n} c_i s_i \leq B \tag{8.1}$$

The companion problem can be defined as follows:

$$\text{minimize} \quad \sum c_i s_i$$

$$\text{subject to} \quad \sum_{i=1}^{n} f_i(s_i) \leq (\geq) \quad G^*.$$

where G^* is a specified performance goal. (The direction of the inequality depends on whether (8.1) is a minimization or a maximization problem). This companion problem can either be solved directly using a generalized Lagrange approach like that used to solve the primal problems, or the primal problem can be solved for a range of budget values and the smallest budget satisfying the performance goal be selected.

Solution of this companion problem will provide planners with a budgeting tool through which they can justify a budget on the basis of the performance that can be attained. Thus it provides a mechanism for relating resources to readiness.

9. SUMMARY

We have developed alternative wholesale provisioning models which are intended for use by both SPCC and ASO. The models determine how to allocate optimally a given provisioning budget with respect to various measures of performance. The measures of effectiveness can be summarized into three types:

1) Essentiality-weighted units/requisitions short or SMA,

2) Essentiality-weighted time-weighted units short or MSRT, and

3) Availability.

In addition to providing the optimal budget allocations, each model provides a prediction of the performance to be expected during the provisioning period.

The models all assume that a budget, as determined by the COSDIF process, is given. However, each model can be converted easily into a related optimization problem which determines the minimum budget required to achieve a specified performance goal. We feel that it makes more sense to determine a budget in this manner, than by using the COSDIF approach which is heavily cost oriented and which provides no estimates of the performance to be expected with a given budget.

The alternative models solve simultaneously the range and the depth problems. Thus, no additional model is required for the budget allocation process.

In order to evaluate the proposed alternative models, the models should be exercised with sample data, and the results examined carefully to observe how each model allocates the provisioning budget. The results of each model should be tested with respect to several measures of performance (units short, SMA, time-weighted units short, MSRT, availability, etc.). One should then select that model which provides results that best agree with a decision maker's perception of how the system should perform. In exercising the alternative models the present models should also be tested to serve as a baseline for comparison. This is the type of model evaluation that will be discussed in the second report.

However, even before a real data performance evaluation is performed, the alternative models can be evaluated theoretically or intuitively to determine if they attempt to achieve the goals of the supply system. We have commented on various properties of each model in the sections developing the models and in the previous chapter. Because of those theoretical properties of the models, and because of the role that time-weighted shortages (MSRT) plays in the expression for availability, we believe that the MSRT model makes the most sense for provisioning. It captures the essence of the objectives of the supply system while avoiding some of the problems and data requirements that would be associated with the pseudo-availability model.

APPENDIX A: INITIAL DEMAND FORECASTING

THE PRESCRIBED PROCEDURE

The initial demand prediction process has been delineated in Reference [7] and begins with a forecast of "program data" from POC out over the two-year demand development period. This data is a month-by-month schedule of the anticipated end item installations over that time period. The time-weighted average months program (TWAMP) is computed next for the appropriate program time base (PTB). Reference [7] allows the PTB to have a maximum length of one year. Shorter PTBs are 3 months and 6 months and are appropriate for computing needs for expensive spares. However, SPCC and ASO use only a 12 months PTB regardless of an item's value because of the workload created by having shorter PTBs (a review is needed at the end of each PTB interval for the shorter ones).

The concept behind the TWAMP is quite simple. Consider Figure A.1; the TWAMP is computed by determining the area under the curve of the installation schedule and then dividing it by the length of the PTB. This results in an average number of end items to be supported over the PTB. This average is denoted as the initial TWAMP or TWAMP by Reference [19].

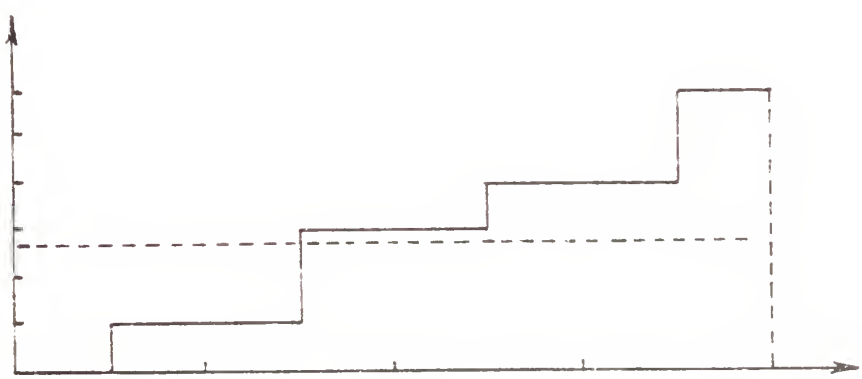

Figure A.1: An Installation Schedule

The following formulas are provided by Reference [7] for computing the initial TWAMP. They obtain the area by summing the vertical slices of the area spanning each month. Any installations occurring in a specific month are assumed to begin operation in the middle of the month.

$$D_m = \begin{cases} \dfrac{I_1}{2} & \text{for } m = 1 \\ \displaystyle\sum_{k=1}^{m-1} I_k + \dfrac{I_m}{2} & \text{for } m \geq 2 \end{cases}$$

where

I_k = the number of units of the end item to be installed in month k.

$$TWAMP_I = \frac{\sum_{m=1}^{PTB} D_m}{PTB} .$$

The initial annual demand rate for a given spare part can be computed by using the following formula [19]:

$$D_A = TWAMP_I * N * BRF$$

where

N = the number of units of a given replaceable part in the end item.

BRF = the "best replacement factor" which is, in fact, the estimated failure rate of one unit over a year.

The BRF is initially based on a technical replacement factor (TRF) which is the contractor's estimate of the attrition rate.

The COSDIF formula requires an initial estimate of the steady state annual demand rate D_{SS}. This is computed by first determining the number of end items to be installed over the first two years (its value is denoted as $TWAMP_{SS}$ in Reference [19]). Then

$$D_{SS} = TWAMP_{SS}*N*BRF.$$

If a spare part is repairable then the formula considers only attrition demand. In that case,

$$D_{SS} = TWAMP_{SS}*N*BRF[1 - CRR*RSR] \qquad (A.1)$$

where:

CRR = the carcass return rate, and

RSR = the repair survival rate.

The initial estimate of demand for repairables will be introduced later.

The development of the provisioning budget constraint of Reference [7] requires that the initial demand during the procurement leadtime (PCLT) plus one quarter be computed. This is then the depth value to be used to develop the budget constraint. Reference [19] also needs the demand during PCLT for the Variable Threshold model. For consumable items the formulas are, respectively,

$$D_{PCLT} = D_A*\frac{PCLT}{4};$$

$$D_{PCLT+1} = D_A*\frac{(PCLT+1)}{4}.$$

For a repairable item the computation used in Reference [19] includes the repair turnaround time (TAT). The formulas are based on the following logic. Without any carcass returns the attrition demand is the same as Equation

(A.1) with $TWAMP_{SS}$ replaced by $TWAMP_I$. If the repair process is "up and running" at POC minus TAT, then the first repaired item will be available at POC and will provide a replacement for some unit of the item when it fails. The net attrition loss is then

$$TWAMP_I * N * BRF * [1 - CRR * RSR]$$

If, as is expected, the repair process can't begin until some items fail, then the net attrition loss just described must be increased by the number of units required to "charge" the repair pipeline assuming that TAT is shorter than the PCLT or PCLT plus one quarter. The respective formulas for demand of repairables are therefore:

$$D_{PCLT} = D_A * [\frac{PCLT}{4} * (1 - CRR * RSR) + \frac{TAT}{4} * CRR * RSR]$$

$$D_{PCLT+1} = D_A * [\frac{(PCLT+1)}{4} * (1 - CRR * RSR) + \frac{TAT}{4} * CRR * RSR]$$

APPENDIX B: THE COSDIF FORMULA

B.1 THE PHILOSOPHY

The COSDIF formula was provided by Alan Kaplan, of the Army's Inventory Research Office, who was a member of the team that developed the DODINST 4140.42 [7]. It is derived from the Army's wholesale range model. It can be viewed as the results of a simple decision model as illustrated in Figure B.1.

Decision	States of Nature: No demand during DDP	States of Nature: Demand during DDP
Make a provisioning buy	Costs of procurement plus two years' holding costs	Two years of average annual variable costs
Do not make a provisioning buy	No costs	Costs of spot buys during first year and average annaul variable costs for second year

Figure B.1: A Provisioning Decision Matrix

The expected costs of each decision can be evaluated when all the costs and the probability values for the states of nature are known. That decision which corresponds to the least expected cost is then the optimal one.

The COSDIF formula given below determines the difference between the expected costs of making a buy and not making a buy.

If the COSDIF value is negative, then the costs associated with not making a buy are greater than those for making the buy, and is therefore optimal to make the buy. If the COSDIF is positive, then making a buy is more costly. If the COSDIF is zero, either decision would be optimal but not making a buy is less work for the provisioner so no buy is made.

$$\begin{aligned} COSDIF = {} & (D_O | D_T) * \{CP + 2*H*C*(R + D_{SS}/FP)\} \\ & + (1 - D_O | D_T) * \{CP*FP + H*C*D_{SS}/(2*FP) + D_T*CI\} \\ & - (1 - D_O | D_T) * \{D_T*(CSP + PLT*\lambda/4) + D_T*C*P\} \end{aligned}$$

where:

D_{SS} = steady state annual demand (attrition)

D_T = total steady state annual demand ($TWAMP_{SS}*BRF$)

$D_O | D_T$ = probability of no demand in two years, given an annual steady state demand forecast (total) of D_T. This value will be obtained from the conditional probabilities stored in the SCA after the computed D_T is rounded up at .5.

CP = cost of procurement [the small purchase value is used if the value of the buy quantity is less than or equal to the small/large purchase breakpoint; if not, the large purchase value is used. The specific values to be compared to the breakpoint are the values of the buy quantity $(R + D_{SS}/FP)$ and the economic order quantity (D_{SS}/FP) for lines 1 and 2, respectively, of the formula].

H = holding cost rate

C = unit price

R = reorder point quantity

FP = frequency of procurement

CI = cost of issuing stock

CSP = cost of spot procurement

PLT = production lead time

λ_S = implied shortage cost [the larger of the shortage cost for COSDIF (specified in the SCA cog constants) and the cost to hold, H*C]

P = spot buy premium rate

In examining the COSDIF formula we see that the second and third lines correspond to the outcomes of the alternative decisions when there is demand during the DDP (see the last column of Figure B.1). However, they show only the average annual variable costs for one year and the expected costs of making spot buys for one year. This is because the average annual variable costs for the second year cancel each other when the cost difference is taken according to [7].

B.2 A CRITIQUE

The COSDIF formula serves as the range model in the development of the provisioning budget constraint. As a consequence it is appropriate to examine its elements and to critique the assumptions. The purpose of this critique is to address certain details of the formula so that the reader will be able to identify where improvements in the current use of the model by the Navy can be made. It is not possible to change the formula without approval, and recent attempts to do so have not been successful. These will be discussed below.

A rather puzzling assumption involves the use of the steady-state demand. It is especially so when we realize that the depth model does not use the steady state value. Perhaps it is true that, because cost differences are being used, use of D_{SS} instead of D_A should make little relative difference. This should be evaluated.

The COSDIF model considers only two states of nature with respect to the occurrence of demands over the two year demand development period--zero demands or positive demands. SPCC uses probability estimates of these states which are a function of the expected mean demands. The probabilities are given in Reference [10]. The probabilities used by SPCC are more appropriate than those in [7] since the latter were based on inventory records for non-Navy items. In fact, [7] required each service to come up with better values within two years. The probabilities used by SPCC can be further improved by developing values for each two-digit cog as was attempted in [10]. This is strongly recommended since cog classes do represent items of a similar nature. It is interesting to note that ASO has apparently continued to use the probability values given in [7] for all of its cog classes [6].

It could be argued that greater resolution in the demand distribution should be considered in the COSDIF expression--perhaps estimates of individual positive demand quantities for, say, quarterly or yearly periods over the two-year DDP. Even if the probability was kept at a single value, the form of the average annual variable costs to stock an item over the two years suggests that several buys will be made and orders received. The buys might be made but, as was noted in the main part of this report, no deliveries are expected until after the end of the DDP. In fact, the average variable cost expressions assume that steady state is reached instantaneously at POC (as the use of D_{SS} implied earlier). A switch from a no-stock to a stocking mode between the first and second year is virtually impossible because of the leadtimes. The spot buy situation will continue through all of the DDP under current replenishment leadtime values. There will be no holding costs since no inventory will be on hand.

The term of the COSDIF equation corresponding to the zero-demand state of nature is the closest to being correct over the two year DDP. The impact of stocking and holding for two years is significant and is not unexpected in light of the concern in [7] about excessive unused stocks of provisioned items.

Considering the spot buy term in more detail, it is interesting to note that there appears to be no explicit term for an item's essentiality. In an earlier document (DODINST 4140.39 [6]), the replenishment model included such a term. If one searches through [7] for a reason why essentiality was not included, the only consideration that is found is in Paragraph IV.E.:

Non-demand-based items, which do not meet the insurance item criteria for wholesale level stockage may be stocked in the wholesale system only if there is an overriding requirement to do so based upon their essentiality in a selected weapon system. In this instance, an item will be stocked as a Numeric Stockage Objective (NSO) item.

Reference [7] also notes that

no deviation to the basic formula is authorized, the use of cost data more relevant to a particular inventory should provide sufficient flexibility to account for the variability required in support of different systems. Data developed for ICP wholesale level systems in accordance with DODINST 4140.39 should be useful in establishing the most appropriate holding costs, procurement costs, and implied shortage cost.

Additionally, a constraint is imposed such that the cost of a requisition on backorder must be at least equal to the cost to hold an equivalent amount of material. It may also be desirable to use a larger value for the holding cost in the COSDIF than is used in Reference [6] to reflect a higher obsolescence probability for marginally stocked items [7].

A variation in the COSDIF which includes an essentiality term was presented in Reference [9]. However, the goal of that reference was to justify the variable threshold model (which does include an essentiality term in the RISK equation used to determine the depth) as an alternative model for implementation of the budget constraint. It is important to note that D55, which is SPCC's implementation of the COSDIF, does not use this variation in creating the budget constraint.

In July 1981, the Office of the Chief of Naval Operations [4] proposed changing Reference [7] to correspond to the form of the COSDIF in [9]. The response from the Deputy Assistant Secretary of Defense (Logistics and Material Management) was [11]:

While we understand the thrust of your initiative to revise DoD Instruction 4140.42, we do not agree with the necessity of making the requested changes. This instruction currently recognizes essentiality in the initial spares computation and allows stockage of essential items even if they do not qualify for stockage through the use of the cost differential tables.

When a shortage cost is considered in any inventory model supporting a weapon system, it is implicitly a measure of the essentiality of the item. The value that can be used for λ_s in the COSDIF equation is not constrained by References [7] and [17] to be that of the replenishment model. In fact, the values used in the replenishment models are tied to a replenishment budget constraint in any given fiscal year. As a consequence, the shortage cost values in the replenishment model can change each year. It should be obvious then that such values are inappropriate for provisioning.

Control over λ_s will have a significant effect on the development of the budget constraint. The larger the λ_s value, the larger will be the range of demand-based items used in developing the budget constraint. The bottom line is to obtain a larger budget constraint. The determination of better λ_s values should be viewed as an important first step of improving the provisioning process. The ASO provisioning model attempts to do this. It adjusts λ_s in an effort to gain an SMA of 85% for the depth allowed by [7] for determining the budget constraint. This is done separately for 1R cog and for the group of 2R and 6R cog items. The ASO model develops a cost constraint for five different values of λ_s using application/operation D54 [17]. For each λ_s, the depths of the items are used to estimate the SMA which would result. A curve is then constructed of SMA versus investment. Additional λ_s values as needed are used to generate additional points as the process zeros in on the 85% SMA point. The investment value for the 85% SMA point is then established as the budget constraint. The actual range and depth of provisioned items are computed using D52 [18]. D52 uses a load list approach developed prior to Reference [7]; the goal of that approach is to minimize expected units short. The λ_s value generated by D54 is used only to generate the budget constraint. D52 uses a Lagrange multiplier approach to implement that budget.

It is debatable whether SMA is the appropriate effectiveness goal to use in light of the current trend toward A_o. Additionally, SMA is measured only for those items passing the COSDIF range criterion. However, the approach used in the ASO model is certainly worth considering with availability goals used in place of the SMA goal.

APPENDIX C

This appendix gives a simple counter-example to show that maximization of "pseudo-availability" as defined by the relationship

$$A_o = \prod_{i}^{n} \frac{MTBF_i}{MTBF_i + MTTR_i + MSRT_i(s_i)}$$

does not yield the same solution to the stockage allocation problem as minimization of mean supply response time (MSRT) or time-weighted units short (TWUS). Thus, the claim that "sparing to availability" is equivalent to sparing to minimize logistics delay time is not true.

Example: Consider a system consisting of two components having the parameter values given in Table C1:

PARAMETER	1	2
MTBF	0.200	0.100
PCLT	1.0	1.0
MTTR	0.0822	0.0274
C	5.	10.

TABLE C1: SYSTEM PARAMETERS

where PCLT is the procurement leadtime in years; C is the unit cost; $MTBF = 1/\lambda$ is the mean time between failures; and MTTR is the mean time to repair. Suppose we are allowed a total provisioning budget of $B = \$20$. Then it is clear that there are only 3 undominated feasible solutions to the allocation problem: (4,0), (2,1), and (0,2). Table C2 gives the mean supply response times for each of the feasible values of stockage levels for components one and two.

MSRT(s)

s / Item	0	1	2	3	4
1	182.50	124.00	79.51	47.80	26.83
2	182.50	149.65	120.45	x	x

TABLE C2: MSRT vs. s

From the information in Tables C1 and C2 it is easy to determine the overall mean supply response time and the system availability for each of the three candidate solutions. These values are given in Table C3.

CANDIDATE SOLUTION	MSRT	A_o
(4,0)	130.61	0.0896*
(2,1)	126.27*	0.0744
(0,2)	141.13	0.0559

TABLE C3: SYSTEM MSRT AND A_o

The values in Table C3 reveal that the solution (2,1) is optimal with respect to total MSRT whereas the solution (4,0) is optimal with respect to system availability!

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